INFLUENCE OF RANDOM BULK INHOMOGENEITIES ON QUASI-OPTICAL CAVITY RESONATOR SPECTRUM

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We suggest the statistical spectral theory of oscillations in quasi-optical cavity resonator filled with random inhomogeneities. It is shown that inhomogeneities in the resonator result in inter-mode scattering leading to the shift and broadening of spectral lines. The shift and broadening of each line essentially depends on frequency distance to adjacent spectral lines. With increasing the distance the influence of inhomogeneities sharply reduces. The solitary spectral lines which have the distance to the nearest lines quite large is slightly changed due to small inhomogeneities. Owing to such selective influence of inhomogeneities on the spectral lines the effective spectrum rarefaction appears. Both the shift and broadening of spectral lines as well as spectrum rarefaction in quasi-optical cavity millimeter wave resonator were detected experimentally. We found out that inhomogeneities result in stochastization of the resonator spectrum in that mixed state appears, i.e. the spectrum acquires both regular and random parts. The active self-oscillator system based on the inhomogeneous quasi-optical cavity millimeter wave resonator with Gunn diode was studied as well. The inhomogeneous quasi-optical cavity millimeter wave resonator (passive and active) can serve as a model of semiconductor quantum billiard. Based on our results we suggest using such billiards with spectrum rarefied by random inhomogeneities as an active system of semiconductor laser.

I. INTRODUCTION

The problem of electromagnetic wave propagation in random inhomogeneous media has been the issue of the day for several decades. Numerous publications are devoted to the analysis of different aspects of this problem (see [1,2] and references therein). In this scientific area the subject of research is normally the scattering of electromagnetic (acoustic) waves in unbounded or partially confined systems (for example, in waveguides) which contain random inhomogeneities. Researches on wave propagation in open statistically unregulated systems are stimulated by numerous applications for long-distance signal transmission in both radio and optical wave ranges.

The problem of electromagnetic oscillations in confined systems with random inhomogeneities has significant meaning as well. At the same time, by now it has not been developed enough both from theoretical and experimental side. It is caused by, in particular, modern theoretical studies of electromagnetic wave propagation and scattering in random inhomogeneous media use assumption about the isotropy and homogeneity scattering medium filled confined systems [1,2]. Therefore, they cannot be applied in general to the study of confined systems such as cavity resonators. The experimental study also requires new approach to study electromagnetic oscillations in inhomogeneous cavity resonators in wide wavelength band.

In our recent paper [3] we suggested a new spectral approach to study confined systems with random inhomogeneous. There spectral properties of cavity spherical quasi-optical millimeter wave resonator filled with random sapphire particles were considered. The sapphire particles with dimensions of the order of operating wavelength affect significantly the resonator spectrum. The spherical frequency degeneration is completely removed and the spectral lines thus have chaotic distribution on the frequency scale. The lines became wider and the quality factor is correspondingly decreased. The analogous broadening of spectral lines caused by random inhomogeneities has been detected in Refs. [4,5], where resonators with random rough boundaries were studied.

Recently the question about the influence of random inhomogeneities on the resonator spectrum has attracted a great attention in relation to the design of lasers on open micro-resonators [6]. In such resonators, the whispering-gallery modes with super-high quality factors can be excited. The possibility to realize such quality factors depends essentially on the number of inhomogeneities (roughness level) on the resonator boundaries.

It should be realized that cavity resonator with extremely small dissipation loss is the almost Hermitian system whose oscillations are formed due to the restricted motion of electromagnetic waves. Therefore, the resonator spectrum is discrete. Random inhomogeneities whether bulk or on the resonator boundaries not introducing additional dissipative loss in general have not to broaden the spectral lines. Nevertheless, the broadening of lines in the spectrum effects and its stochastization have been detected in experiment [3].

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In this connection the question arises: what is the physical mechanism of spectral lines broadening and spectrum stochastization in the random inhomogeneous resonator? The study of this question is one of the reasons of the present paper. Another motivation is relevant to the nanoelectonics problem. Recently the great emphasis is the study of a new type of nanoelectron systems (so-called zero-dimensional ones). The charge carrier motion in these systems has space restriction in all three dimensions. The peculiar example of zero-dimensional system is a quantum dot (QD). QD is a semiconductor area by the size of order of 10 nm with electron (hole) conductivity and restricted from outer area by potential barrier. Owing to finite charge carrier motion the energy spectrum in QD is discrete and the number of spectral levels due to small QD size is relatively few.

Recently an elegant method of QD array implementation has been designed. This method is based on the self-organization effect in strained double GaAs heterostructures [7-9]. The usage of QD array as an active medium permits to design lasers with high performance [10-12]. However, the self-organization process of QD array forming is difficult to manage. The random inhomogeneities that usually exist in heterostructure essentially affect on this process. They lead to inhomogeneous broadening of spectral lines [13] and, correspondingly, to degradation of laser radiation quality.

The QD design method based on the self-organization effect is, in fact, an alternative to electron lithography whose level of development does not permit to implement the ordered array of approximately the same QDs with small enough dispersion of their sizes. In the present paper we propose another way to design semiconductor laser system. This way presupposes the usage of the same regular microscopic areas ordered array that can be implemented by lithography as an active laser medium. This proposal is based on the following. At present technology of GaAs mono-crystal with super high mobility and big length of phase coherence of charge carriers that achieves 10 μm and more is well-developed. Due to that the system of microscopic regular areas with potential barrier on the margin of each of them made from such materials can be implemented. Because of big length of phase coherence, carriers will take part in ballistic motion and reflect back from area margins. The motion of charge carriers is similar to dynamics of billiard systems. Since this motion is described by Shrodinger equation such a billiard system can be characterized as a quantum billiard (QB).

Due to finite motion of charge carrier, electron spectrum of QB is discrete. At the same time it is quite dense, because of a QB size is much bigger than the wavelength of quasi-particles. This fact makes complicate the QB usage as an active system for the semiconductor laser, because dense spectrum decreases frequency stability of laser radiation. The frequency jumps appear easily at the small deviation of control parameters in the laser resonator with dense frequency spectrum. As a result there is a problem of "rarefaction" of the spectrum: *can the dense QB spectrum be done much sparser without changing of QB geometrical parameters?*

A quasi-optical cavity resonator has dense and discrete frequency spectrum as well. The Maxwell equation describing electromagnetic oscillations in such a resonator coincides with corresponding scalar Shrodinger equation at definite conditions. All that gives an opportunity to use the quasi-optical resonators as model objects to study spectral properties of QB. Similar modeling using microwave resonators was done earlier for study of the phenomena that is relevant to quantum chaos [14-17].

In the present paper the nature of spectral lines broadening and their shifting are studied in a quasi-optical millimeter wave cavity resonator with random inhomogeneous inside. To clarify the mechanism of broadening and shifting the original statistical spectral theory based on the separation mode technique was worked out. This technique was designed earlier by one of the author for open waveguide systems [18,19]. Using this technique we found out that the mechanism of spectral lines broadening was caused by intermode scattering. At that the largest broadening are subjected to closely set lines. And solitary lines keep high quality factor and intensity in the random inhomogeneous resonator. It results in so-called spectral "rarefaction".

The systematic experimental spectral measurements of numerous realizations of quasi-optical random inhomogeneity cavity resonator filling were carried out for theory results implementation. The experimental results proved that the main mechanism of broadening and shifting of spectral line is inter-mode scattering caused by inhomogeneities resonator filling. We detected theoretically predicted effect of "rarefaction" of the resonator spectrum. Thus, we determined that the influence of inhomogeneities on the resonator spectrum has both, as known, negative nature (broaden lines) and a positive one. This positive influence is essential "rarefaction" of the spectrum. This fact can be important in QB usage as an active system in a semiconductor laser approach. Here we state that the high quality modes are not subjected, practically, to broadening and can satisfy self-excitation laser condition even under big inhomogeneities in an active resonator medium. Exactly the stable laser generation can be implemented under one of such modes.

Another aspect of influence of inhomogeneities on resonator spectrum relates to its stochastization. In the paper we carried out statistical analysis of the resonator spectrum with a different number of inhomogeneities. We found out that the inter-frequency (IF) interval distribution in the resonator spectrum is close to Poisson distribution in a small number of inhomogeneities. This kind of distribution has systems with non-correlated IF intervals. The stochastic spectrum part appears at the increase of the number of inhomogeneities. The spectrum is mixed, i.e. contains regular and stochastic parts. We estimated relationship between regular and stochastic spectrum parts by statistical distribution analysis of IF
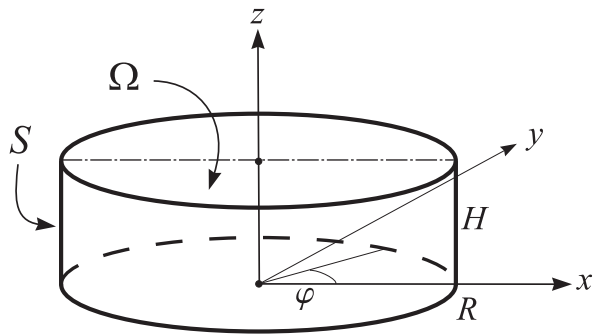
FIG. 1: The geometry of the cylindrical quasi-optical cavity resonator. S is the resonator side face, Ω is its volume, H is the height of the cylinder, R is the radius of its base.

intervals depending on the quantity of inhomogeneities in the resonator. We carried out the modeling of semiconductor laser based on QB with random inhomogeneities. For the purpose of this we used a quasi-optical millimeter wave cavity resonator filled with random inhomogeneities and with an inserted Gunn diode as an active element. In such a self-oscillation system we studied self-excitation oscillation conditions. We found out that the generation is unstable and multi-frequency exists in an empty resonator near the excitation threshold because of dense spectrum. Such generation can be explained by frequency jumps in dense resonator spectrum. These jumps disappear because of "rarefaction" of the spectrum if inhomogeneities are inserted into the resonator, and total number of generating frequencies is decreased at a definite range of control parameters deviation.

II.   STATISTICAL SPECTRAL THEORY OF THE RESONATOR WITH RANDOM BULK INHOMOGENEITIES.

A.   Statement of the problem

Let us consider a cylindrical quasi-optical cavity resonator of radius R and height H (see Fig.1). Inner volume of the resonator, Ω, is assumed to be filled with the material having random inhomogeneous permittivity. We will be interested in oscillations that turn into transverse-electrical resonance mode (TE-mode) provided that the resonator is filled homogeneously. The vertical (z) component of the electrical field of this mode is equal to zero.

According to Ref. [20], electromagnetic field of the TE mode can be calculated trough magnetic Hertz vector having only one non-zero component, namely, z-component $\Pi_z(\mathbf{r})$. We assume that inhomogeneity of permittivity of the resonator infill is small. In this case to define $\Pi_z(\mathbf{r})$ in the inhomogeneous resonator we can use

the approximate wave equation

$$\left[\Delta + k^2 \varepsilon(\mathbf{r})\right] \Pi_z(\mathbf{r}) = 0 , \qquad (1)$$

where all components of the Hertz vector, except z-component, are assumed of zero value taking into account small inhomogeneity. In Eq.(1), Δ is the three-dimensional (3D) Laplacian, $\varepsilon(\mathbf{r}) = \varepsilon_0 + \delta\varepsilon(\mathbf{r}) + i\alpha$ is the complex permittivity whose imaginary part α takes phenomenologically into account ohmic loss in the system; the function $\delta\varepsilon(\mathbf{r})$ describes random space fluctuations of the permittivity around its average value ε_0, $k = \omega/c$ is wavenumber.

In this paper we are interested in the analogy between electromagnetic resonators and solid-state microobjects, namely, QDs and QBs. In the case of classical resonance system, the problem of excitation by a given point monochromatic source is governed by the equation Eq.(1) which should be supplied with δ-term in the right-hand side. The equation thus obtained coincides in form with the equation for Green function of quantum particles moving in dissipative medium and being subjected to inhomogeneous scalar potential. Having this in mind we will carry out all further analysis relevant to the electromagnetic resonator in terms of the dynamic equation for particles in the quantum dot of cylindrical shape.

The equation for Green function of quantum particles moving in dissipative medium under the influence of random static potential $V(\mathbf{r})$ has the form

$$\left[\Delta + k^2 - i/\tau_d - V(\mathbf{r})\right] G(\mathbf{r}, \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}') . \qquad (2)$$

Here τ_d is the dissipative attenuation time whose inverse value has the same physical meaning as the imaginary part of function $\varepsilon(\mathbf{r})$ in Eq.(1) taken with minus sign. The potential $V(\mathbf{r})$ in the case of electromagnetic system is given by $V(\mathbf{r}) = -k^2 \delta\varepsilon(\mathbf{r})$. Boundary conditions for the solution to Eq.(2) result from the requirement of vanishing tangential components of the electrical field of TE mode on the resonator interface. At side boundary S, the Neuman condition

$$\left. \frac{\partial G(\mathbf{r}, \mathbf{r}')}{\partial r} \right|_S = 0, \qquad (3a)$$

should be met whereas at end surfaces $z = \pm H/2$ the Dirichlet condition

$$G(\mathbf{r}, \mathbf{r}')\Big|_{z=\pm H/2} = 0 \qquad (3b)$$

should be satisfied.

For studying oscillations spectrum of the resonator with random inhomogeneities, the poles of Green function averaged over realizations of the potential $V(\mathbf{r})$ from Eq.(2) should be determined. This function can be found, for example, from Dyson equation. Yet there by now do not exist effectual methods for solving this equation in the case of confined multi-dimensional systems. In this paper, we apply for this purpose the original calculation

technique which relies on precise separation of modes in an arbitrary confined system, including the disordered one. The method of mode separation was previously developed for solving transport problems in disordered 2D open systems [18] and then modified for systems of waveguide geometry in three dimensions [19, 21, 22]. Below we set forth adaptation of this technique for systems of closed geometry, in particular, for cavity resonators and quantum dots.

Let us at first turn to mode representation of the equation Eq.(2) using some set of basis functions. The most appropriate for our purposes seems to be the whole set of eigenfunctions of the Laplace operator. For the cylindrical resonator shown in Fig.1 these functions can be factorized to the form

$$|\mathbf{r}, \boldsymbol{\mu}\rangle = |r, \varphi; l, n\rangle |z, q\rangle \ , \tag{4}$$

where $\mathbf{r} = (r, \varphi, z)$ is the radius-vector in cylindrical coordinates, $\boldsymbol{\mu} = (l, n, q)$ is the vectorial mode index conjugate to that vector. Normalized eigenfunctions of "transverse" part of the Laplacian, which obey boundary conditions Eq.(3a), are given by

$$|r, \varphi; l, n\rangle = C_{ln}/\left(\sqrt{\pi}R\right) J_{|n|}\left(\gamma_l^{(|n|)} r/R\right) e^{in\varphi} \tag{5a}$$

$$l = 1, 2, \ldots, \quad n = 0, \pm 1, \pm 2, \ldots, \, ,$$

where the coefficient C_{ln} has the form

$$C_{ln} = \frac{\gamma_l^{(|n|)}}{\left[\left(\gamma_l^{(|n|)}\right)^2 - n^2\right]^{1/2} J_{|n|}\left(\gamma_l^{(|n|)}\right)} \ . \tag{5b}$$

The set of coefficients $\gamma_l^{(|n|)}$ in Eqs.(5) consists of positive zeros of the function $J'_{|n|}(t)$ which are numbered by index l in ascending order. The eigenvalues corresponding to functions Eq.(5a) are equal to $\xi_{ln} = -\left(\gamma_l^{(|n|)}/R\right)^2$. Basis functions of the "longitudinal" Laplacian, $\partial^2/\partial z^2$, which meet boundary conditions Eq.(3b), are given by

$$|z; q\rangle = \sqrt{\frac{2}{H}} \sin\left[\left(\frac{z}{H} + \frac{1}{2}\right)\pi q\right] \tag{6}$$

$$q = 1, 2, \ldots \ ,$$

the corresponding eigenvalues being equal to $(\pi q/H)^2$.

In the basis of functions (4), Eq.(2) takes the form

$$\left(k^2 - \kappa_{\boldsymbol{\mu}}^2 - i/\tau_d - \mathcal{V}_{\boldsymbol{\mu}}\right) G_{\boldsymbol{\mu}\boldsymbol{\mu}'} - \sum_{\boldsymbol{\nu} \neq \boldsymbol{\mu}} \mathcal{U}_{\boldsymbol{\mu}\boldsymbol{\nu}} G_{\boldsymbol{\nu}\boldsymbol{\mu}'} = \delta_{\boldsymbol{\mu}\boldsymbol{\mu}'} \ . \tag{7}$$

Here, $G_{\boldsymbol{\mu}\boldsymbol{\mu}'}$ is the Green function in mode representation, the parameter

$$\kappa_{\boldsymbol{\mu}}^2 = \left(\frac{\gamma_l^{(|n|)}}{R}\right)^2 + \left(\frac{\pi q}{H}\right)^2 \tag{8}$$

is the unperturbed "energy" of the mode $\boldsymbol{\mu}$ (the eigenvalue of 3D Laplace operator), functions $\mathcal{U}_{\boldsymbol{\mu}\boldsymbol{\nu}}$ are mode matrix elements of the random potential,

$$\mathcal{U}_{\boldsymbol{\mu}\boldsymbol{\nu}} = \int_\Omega d\mathbf{r} \, \langle \mathbf{r}; \boldsymbol{\mu} | \, V(\mathbf{r}) \, | \mathbf{r}; \boldsymbol{\nu} \rangle \tag{9}$$

Attention should be drawn to the fact that the *intramode*, i.e. diagonal in mode indices, matrix element $\mathcal{U}_{\boldsymbol{\mu}\boldsymbol{\mu}} \equiv \mathcal{V}_{\boldsymbol{\mu}}$ is separated in Eq.(7) from other terms of the sum where thus only matrix elements corresponding to *intermode* scattering are left. It was shown in Ref. [18] that such a separation of intra- and intermode effective potentials provides mathematical correctness of the derivation of closed equations for the diagonal components of Green matrix $\|G_{\boldsymbol{\mu}\boldsymbol{\mu}'}\|$ and through those components for that matrix integrally.

B. Separation of the modes

To solve the infinite set of coupled equations Eq.(7) is not less intricate problem than direct solution of multidimensional differential equation Eq.(2). The problem would be resolved elementary if the resonator modes allowed for their strict separation. Normally, modes are easily separable if the resonator has no inhomogeneities. It will be shown below, based on the technics developed in Refs. [18, 19, 21], that in fact they can be separated even in the case of arbitrarily inhomogeneous resonator. But the cost of this separation in the general case is the appearance in equations for each of the modes of the effective potential, known as T-matrix in quantum theory of scattering [23], whose functional structure is much more involved than that of the initial potential $V(\mathbf{r})$.

As a starting point for mode separation we introduce unperturbed (or trial) mode propagator $G_{\boldsymbol{\nu}}^{(V)}$ by omitting in Eq.(7) all intermode potentials $\mathcal{U}_{\boldsymbol{\mu}\boldsymbol{\nu}}$,

$$G_{\boldsymbol{\nu}}^{(V)} = \left(k^2 - \kappa_{\boldsymbol{\nu}}^2 - i/\tau_d - \mathcal{V}_{\boldsymbol{\nu}}\right)^{-1} \tag{10}$$

The term "unperturbed" will thus be used hereupon with respect to intermode potentials, intramode ones being accounted for precisely.

By substituting $\boldsymbol{\mu}' = \boldsymbol{\mu}$ in Eq.(7) we obtain linear nonuniform connection of intramode propagator $G_{\boldsymbol{\mu}\boldsymbol{\mu}}$ with all intermode Green functions having the particular right-hand mode index $\boldsymbol{\mu}$,

$$G_{\boldsymbol{\mu}\boldsymbol{\mu}} = G_{\boldsymbol{\mu}}^{(V)}\left(1 + \sum_{\boldsymbol{\nu} \neq \boldsymbol{\mu}} \mathcal{U}_{\boldsymbol{\mu}\boldsymbol{\nu}} G_{\boldsymbol{\nu}\boldsymbol{\mu}}\right) \ . \tag{11}$$

Assuming then $\boldsymbol{\mu}' \neq \boldsymbol{\mu}$ and performing some necessary re-labellings of mode indices we can reduce Eq.(7) to the form

$$\left[G_{\boldsymbol{\nu}}^{(V)}\right]^{-1} G_{\boldsymbol{\nu}\boldsymbol{\mu}} - \sum_{\substack{\boldsymbol{\nu}' \neq \boldsymbol{\nu} \\ \boldsymbol{\nu}' \neq \boldsymbol{\mu}}} \mathcal{U}_{\boldsymbol{\nu}\boldsymbol{\nu}'} G_{\boldsymbol{\nu}'\boldsymbol{\mu}} = \mathcal{U}_{\boldsymbol{\nu}\boldsymbol{\mu}} G_{\boldsymbol{\mu}\boldsymbol{\mu}} \tag{12}$$

$$(\boldsymbol{\nu} \neq \boldsymbol{\mu}) \ .$$

The latter system of interconnected equations can be solved with respect to intermode elements of the Green matrix. In the operator form the solution is given by

$$G_{\boldsymbol{\nu\mu}} = \hat{\mathbf{P}}_{\boldsymbol{\nu}} \left(1 - \hat{R}\right)^{-1} \hat{R} \hat{\mathbf{P}}_{\boldsymbol{\mu}} G_{\boldsymbol{\mu\mu}} \ , \qquad (13)$$

where the linear operator $\hat{R} = \hat{G}^{(V)}\hat{\mathcal{U}}$ of intermode scattering is introduced, which acts in mode subspace $\overline{M}_{\boldsymbol{\mu}}$ consisting of the whole set of mode indices but the index $\boldsymbol{\mu}$; $\hat{\mathbf{P}}_{\boldsymbol{\mu}}$ is the projection operator whose action reduces to the assignment of the value $\boldsymbol{\mu}$ to the nearest mode index of any adjacent operator, no matter standing to the left or to the right of $\hat{\mathbf{P}}_{\boldsymbol{\mu}}$. Operators $\hat{G}^{(V)}$ and $\hat{\mathcal{U}}$ are specified on $\overline{M}_{\boldsymbol{\mu}}$ by matrix elements

$$\langle \boldsymbol{\nu}| \, \hat{G}^{(V)} \, |\boldsymbol{\nu}'\rangle = G_{\boldsymbol{\nu}}^{(V)} \delta_{\boldsymbol{\nu\nu}'} \ , \qquad (14a)$$

$$\langle \boldsymbol{\nu}| \hat{\mathcal{U}} \, |\boldsymbol{\nu}'\rangle = \mathcal{U}_{\boldsymbol{\nu\nu}'} \ . \qquad (14b)$$

Correspondingly, matrix elements of the operator $\hat{R}$ are given by

$$\langle \boldsymbol{\nu}| \, \hat{R} \, |\boldsymbol{\nu}'\rangle = G_{\boldsymbol{\nu}}^{(V)} \mathcal{U}_{\boldsymbol{\nu\nu}'} \ . \qquad (15)$$

By substituting intermode propagators in the form Eq.(13) into the relationship Eq.(11) we obtain the following rigorous expression for intramode propagator $G_{\boldsymbol{\mu\mu}}$,

$$G_{\boldsymbol{\mu\mu}} = \left(k^2 - \kappa_{\boldsymbol{\mu}}^2 - i/\tau_d - \mathcal{V}_{\boldsymbol{\mu}} - \mathcal{T}_{\boldsymbol{\mu}}\right)^{-1} \ . \qquad (16)$$

Here

$$\mathcal{T}_{\boldsymbol{\mu}} = \hat{\mathbf{P}}_{\boldsymbol{\mu}}\hat{\mathcal{U}} \left(1 - \hat{R}\right)^{-1} \hat{R} \hat{\mathbf{P}}_{\boldsymbol{\mu}} \qquad (17)$$

is the portion of the mode $\boldsymbol{\mu}$ eigen-energy which relates to the intermode scattering.

It should be noted that for determining the disordered resonator spectrum the poles of solely diagonal elements of the Green matrix suffice to be found. Just these elements determine, in view of relationship Eq.(13), all major analytical properties of the whole Green function from Eq.(2). Below we will analyze the cavity-resonator spectrum in frames of relatively simple statistical model of random potential $V(\mathbf{r})$.

C. Statistical analysis of the resonator spectrum

We assume the potential $V(\mathbf{r})$ to have zero mean value, $\langle V(r)\rangle = 0$, and the binary correlation function

$$\langle V(\mathbf{r})V(\mathbf{r}')\rangle = DW(\mathbf{r} - \mathbf{r}') \ . \qquad (18)$$

Bearing in mind the forthcoming numerical analysis we will take the function $W(\mathbf{r})$ in the form of Gaussian exponent, viz. $W(\mathbf{r}) = \exp(-\mathbf{r}^2/2r_c^2)$, where r_c stands for the correlation radius. In the case of electromagnetic resonator the normalization constant D in Eq.(18) is given by $D = k^4\sigma^2$, where $\sigma^2 = \langle \delta\varepsilon^2(\mathbf{r})\rangle$ is the variance of permittivity fluctuations.

The pair of selected statistical parameters, namely, the average random potential and its binary correlation function, are sufficient for making detailed analysis of the system in study if the function $\delta\varepsilon(\mathbf{r})$ is the Gaussian-distributed random variable. Yet these two parameters suffice for carrying out the asymptotically correct analysis even in the case where statistics of the fluctuations is markedly non-Gaussian, provided that the potential $V(\mathbf{r})$ is in a certain sense small. As it is conventional in condensed matter physics, we will regard the potential to be small and the resulting scattering, correspondingly, weak provided that the scattering rate calculated in Born approximation is small as compared to the unperturbed quasiparticle energy, k^2 in our case. The smallness of one-fold scattering probability enables one to regard the potential $V(\mathbf{r})$ with parametric accuracy as Gaussian random process, whatever in fact its distribution is [24].

Consider now the self-energy operator of the mode $\boldsymbol{\mu}$ which consists, in accord with Eq.(16), of two terms,

$$\Sigma_{\boldsymbol{\mu}} = V_{\boldsymbol{\mu}} + \mathcal{T}_{\boldsymbol{\mu}} \ . \qquad (19)$$

The first term in the right-hand side of this formula vanishes when being averaged whereas the second term does not. Its average value can be easily calculated under the presumption of weak scattering. The strength of the intermode scattering is estimated by the norm of the operator $\hat{R}$ entering Eq.(17). Assuming this norm to be small as compared to unity and keeping only two main terms in the expansion of the inverse operator in Eq.(17) we come to the necessity of performing the averaging not of the exact operator potential $\mathcal{T}_{\boldsymbol{\mu}}$ but rather of its relatively compact limiting value

$$\mathcal{T}_{\boldsymbol{\mu}} \approx \hat{\mathbf{P}}_{\boldsymbol{\mu}}\hat{\mathcal{U}}\hat{G}^{(V)}\hat{\mathcal{U}}\hat{\mathbf{P}}_{\boldsymbol{\mu}} = \sum_{\boldsymbol{\nu}\neq\boldsymbol{\mu}} \mathcal{U}_{\boldsymbol{\mu\nu}} G_{\boldsymbol{\nu}}^{(V)} \mathcal{U}_{\boldsymbol{\nu\mu}} \ . \qquad (20)$$

When calculating the quantity $\langle \mathcal{T}_{\boldsymbol{\mu}}\rangle$, one can neglect with parametric accuracy the correlation between intramode and intermode potentials. This enables us to average intramode propagator $G_{\boldsymbol{\nu}}^{(V)}$ and its envelopes consisting of intermode potentials $\mathcal{U}_{\boldsymbol{\mu\nu}}$ independently. To average the function $G_{\boldsymbol{\nu}}^{(V)}$ it is worthwhile to present it, at first, in the integral form,

$$G_{\boldsymbol{\nu}}^{(V)} = \int_0^\infty dt \, \exp\left[-i\left(k^2 - \kappa_{\boldsymbol{\nu}}^2 - i/\tau_d - \mathcal{V}_{\boldsymbol{\nu}}\right)t\right] \ . \qquad (21)$$

Then the averaging of the integrand in Eq.(21) with the aid of continual integration with Gaussian functional weight yields

$$\langle G_{\boldsymbol{\nu}}^{(V)} \rangle = \frac{1}{k^2} \int_0^\infty dt \, \exp\left[-i\left(1 - \kappa_{\boldsymbol{\nu}}^2/k^2 - i/k^2\tau_d\right)t - \frac{t^2}{2}\sigma^2 L_{\boldsymbol{\nu}}(r_c)\right]$$

$$= \frac{1}{k^2} \sqrt{\frac{\pi}{2\sigma^2 L_{\boldsymbol{\nu}}(r_c)}} \exp\left[-\frac{\left(k^2 - \kappa_{\boldsymbol{\nu}}^2 - i/\tau_d\right)^2}{2k^4\sigma^2 L_{\boldsymbol{\nu}}(r_c)}\right] \left\{1 - \Phi\left[\frac{i\left(k^2 - \kappa_{\boldsymbol{\nu}}^2 - i/\tau_d\right)}{\sqrt{2k^4\sigma^2 L_{\boldsymbol{\nu}}(r_c)}}\right]\right\} . \tag{22}$$

Here $\Phi(\xi)$ is the probability integral [25], $L_\nu(r_c)$ is the dimensionless correlator of intramode potential $\mathcal{V}_{\boldsymbol{\nu}}$. In the case of Gaussian correlation function $W(\mathbf{r})$ we readily obtain

$$L_{\boldsymbol{\nu}}(r_c) = \frac{1}{k^4} \langle \mathcal{V}_{\boldsymbol{\nu}}\mathcal{V}_{\boldsymbol{\nu}} \rangle = \iint_\Omega d\mathbf{r}\,d\mathbf{r}' \, \langle \mathbf{r}, \nu \,|\mathbf{r}, \nu\rangle \exp\left[-(\boldsymbol{r} - \boldsymbol{r}')^2/2r_c^2\right] \langle \mathbf{r}', \boldsymbol{\nu} \,|\mathbf{r}', \boldsymbol{\nu}\rangle$$

$$= \frac{8}{\pi} C_{l_\nu n_\nu}^4 \int_0^1 \int_0^1 ds\,ds' \exp\left[-\frac{H^2}{2r_c^2}(s - s')^2\right] \sin^2(\pi q_{n_\nu} s)\sin^2(\pi q_{n_\nu} s')$$

$$\times \int_0^1 \int_0^1 tt'\,dt\,dt' J_{|n_\nu|}^2(\gamma_{l_\nu}^{|n_\nu|}t) J_{|n_\nu|}^2(\gamma_{l_\nu}^{|n_\nu|}t') \oint d\varphi \exp\left[-\frac{R^2}{2r_c^2}\left(t^2 + t'^2 - 2tt'\cos\varphi\right)\right] . \tag{23}$$

Although the expression Eq.(22) is formally exact, it is not quite convenient for the analysis in view of its bulky structure. Asymptotical calculations at large and small values of the probability integral argument permit us to make use of much simpler interpolation expression,

$$\langle G_{\boldsymbol{\nu}}^{(V)} \rangle \approx \left(k^2 - \kappa_{\boldsymbol{\nu}}^2 - i/\tau_{\boldsymbol{\nu}}^*\right)^{-1} , \tag{24}$$

which is close in form to the initial unperturbed Green function Eq.(10). Here we have used the notation $1/\tau_\nu^* = 1/\tau_d + k^2\sqrt{(2/\pi)\sigma^2 L_\nu(r_c)}$ for the effective scattering frequency. The latter includes both the initial dissipative term $1/\tau_d$ and the addendum originating from wave scattering by random inhomogeneities.

Interpolation of the average trial Green function by the expression Eq.(24) permits us to interpret the quantity $k^2\sqrt{(2/\pi)\sigma^2 L_\nu(r_c)}$ as the dephasing frequency of the mode state $\boldsymbol{\nu}$, which is related to scattering by the inhomogeneities in the resonator.

To make further comparison of theoretical results with experimental data we will consider below the specific limiting case where the inequalities are fulfilled

$$r_c \lesssim H \ll R . \tag{25}$$

These restrictions enable us to calculate the integrals in Eq.(23) asymptotically, thus resulting in the following estimate for the parameter $L_\nu(r_c)$: $L_\nu(r_c) \sim (r_c/R)^2$. The correlator of intermode potentials in Eq.(20) can be represented as

$$\langle U_{\boldsymbol{\mu\nu}}U_{\boldsymbol{\nu\mu}} \rangle = k^4\sigma^2 \left(\frac{2}{\pi}\right)^2 \int_0^1 \int_0^1 ds\,ds' \exp\left[-\frac{H^2(s - s')^2}{2r_c^2}\right] \sin(\pi q_{n_{\boldsymbol{\mu}}} s)\sin(\pi q_{n_{\boldsymbol{\mu}}} s')\sin(\pi q_{n_{\boldsymbol{\nu}}} s)\sin(\pi q_{n_{\boldsymbol{\nu}}} s')$$

$$\times \int_0^1 \int_0^1 tt'\,dt\,dt' J_{|n_{\boldsymbol{\mu}}|}(\gamma_{l_{\boldsymbol{\mu}}}^{(|n_{\boldsymbol{\mu}}|)}t) J_{|n_{\boldsymbol{\mu}}|}(\gamma_{l_{\boldsymbol{\mu}}}^{(|n_{\boldsymbol{\mu}}|)}t') J_{|n_{\boldsymbol{\nu}}|}(\gamma_{l_{\boldsymbol{\nu}}}^{(|n_{\boldsymbol{\mu}}|)}t) J_{|n_{\boldsymbol{\nu}}|}(\gamma_{l_{\boldsymbol{\nu}}}^{(|n_{\boldsymbol{\mu}}|)}t')$$

$$\times \oint \oint d\varphi\,d\varphi' \exp\left\{-i(n_{\boldsymbol{\mu}} - n_{\boldsymbol{\nu}})(\varphi - \varphi') - \frac{R^2}{2r_c^2}\left[t^2 + t'^2 - 2tt'\cos(\varphi - \varphi')\right]\right\} . \tag{26}$$

Asymptotic calculation of the integrals over φ and φ' results in the following formula

$$\langle \mathcal{U}_{\boldsymbol{\mu}\boldsymbol{\nu}} \mathcal{U}_{\boldsymbol{\nu}\boldsymbol{\mu}} \rangle = k^4 \sigma^2 \frac{r_c}{R} A_{\boldsymbol{\mu}\boldsymbol{\nu}}(r_c) , \qquad (27)$$

where the factor $A_{\boldsymbol{\mu}\boldsymbol{\nu}}(r_c)$ is given by

$$A_{\boldsymbol{\mu}\boldsymbol{\nu}}(r_c) = \frac{8}{\sqrt{\pi}} C^2_{l_{\boldsymbol{\mu}} n_{\boldsymbol{\mu}}} C^2_{l_{\boldsymbol{\nu}} n_{\boldsymbol{\nu}}} \int\limits_0^1 \int\limits_0^1 ds\, ds' \exp\left[-\frac{H^2}{2r_c^2}(s - s')^2 \right] \sin\left(\pi q_{n_{\boldsymbol{\mu}}} s \right) \sin\left(\pi q_{n_{\boldsymbol{\mu}}} s' \right) \sin\left(\pi q_{n_{\boldsymbol{\nu}}} s \right) \sin\left(\pi q_{n_{\boldsymbol{\nu}}} s' \right)$$

$$\times \int\limits_0^1 \int\limits_0^1 \sqrt{tt'}\, dt\, dt' \exp\left[-\frac{R^2}{2r_c^2}(t - t')^2 - \frac{r_c^2 (n_{\boldsymbol{\mu}} - n_{\boldsymbol{\nu}})^2}{4R^2 tt'} \right]$$

$$\times J_{|n_{\boldsymbol{\mu}}|}\left(\gamma_{l_{\boldsymbol{\mu}}}^{(|n_{\boldsymbol{\mu}}|)} t \right) J_{|n_{\boldsymbol{\mu}}|}\left(\gamma_{l_{\boldsymbol{\mu}}}^{(|n_{\boldsymbol{\mu}}|)} t' \right) J_{|n_{\boldsymbol{\nu}}|}\left(\gamma_{l_{\boldsymbol{\nu}}}^{(|n_{\boldsymbol{\nu}}|)} t \right) J_{|n_{\boldsymbol{\nu}}|}\left(\gamma_{l_{\boldsymbol{\nu}}}^{(|n_{\boldsymbol{\nu}}|)} t' \right) . \qquad (28)$$

With the results (24) and (27), separation of real and imaginary parts of the average T-matrix, $\langle \mathcal{T}_{\boldsymbol{\mu}} \rangle = \Delta k_{\boldsymbol{\mu}}^2 + i/\tau_{\boldsymbol{\mu}}^{(ch)}$, results in the following expressions for shifting and broadening the $\boldsymbol{\mu}$-th resonant level,

$$\Delta k_{\boldsymbol{\mu}}^2 = k^4 \sigma^2 \frac{r_c}{R} \sum_{\boldsymbol{\nu} \neq \boldsymbol{\mu}} A_{\boldsymbol{\mu}\boldsymbol{\nu}}(r_c) \mathrm{Re}\langle G_{\boldsymbol{\nu}}^{(V)} \rangle , \qquad (29a)$$

$$\frac{1}{\tau_{\boldsymbol{\mu}}^{(ch)}} = k^4 \sigma^2 \frac{r_c}{R} \sum_{\boldsymbol{\nu} \neq \boldsymbol{\mu}} A_{\boldsymbol{\mu}\boldsymbol{\nu}}(r_c) \mathrm{Im}\langle G_{\boldsymbol{\nu}}^{(V)} \rangle . \qquad (29b)$$

The factor $A_{\boldsymbol{\mu}\boldsymbol{\nu}}(r_c)$, as can be seen from Eq.(28), is a real-valued quantity, its absolute value being estimated as $r_c/R \ll 1$. Although the sign of this factor cannot be uniquely identified in the general case since it contains the dependence on specific indices of modes between which the scattering is carried out, numerical analysis shows that $A_{\boldsymbol{\mu}\boldsymbol{\nu}}(r_c) > 0$, which corresponds to the broadening of spectral lines. One can adequately estimate both the shift and the broadening of $\boldsymbol{\mu}$-th resonant level by substituting the function $\langle G_{\boldsymbol{\nu}}^{(V)} \rangle$ in the interpolated form Eq.(24), instead of its exact expression Eq.(22), into the right-hand sides of formulas Eq.(29). The result in this case reduces to

$$\Delta \kappa_{\boldsymbol{\mu}}^2 = k^4 \sigma^2 \frac{r_c}{R} \sum_{\boldsymbol{\nu} \neq \boldsymbol{\mu}} A_{\boldsymbol{\mu}\boldsymbol{\nu}}(r_c) \frac{\kappa_{\boldsymbol{\mu}}^2 - \kappa_{\boldsymbol{\nu}}^2}{\left(\kappa_{\boldsymbol{\mu}}^2 - \kappa_{\boldsymbol{\nu}}^2 \right)^2 + (1/\tau_{\boldsymbol{\nu}}^*)^2} , \qquad (30a)$$

$$\frac{1}{\tau_{\boldsymbol{\mu}}^{(ch)}} = k^4 \sigma^2 \frac{r_c}{R} \sum_{\boldsymbol{\nu} \neq \boldsymbol{\mu}} A_{\boldsymbol{\mu}\boldsymbol{\nu}}(r_c) \frac{1/\tau_{\boldsymbol{\nu}}^*}{\left(\kappa_{\boldsymbol{\mu}}^2 - \kappa_{\boldsymbol{\nu}}^2 \right)^2 + (1/\tau_{\boldsymbol{\nu}}^*)^2} . \qquad (30b)$$

The structure of the summands in Eqs.(30) indicates unambiguously that both the shift and the broadening of each given resonance ($k^2 \simeq k_{\boldsymbol{\mu}}^2$) are mainly provided by its interactions with the adjacent resonances, whose position on the frequency axis are confined to the region limited by order equality $\left| \kappa_{\boldsymbol{\mu}}^2 - \kappa_{\boldsymbol{\nu}}^2 \right| \sim 1/\tau_{\boldsymbol{\nu}}^*$. In the case where only one resonance level with, say, frequency $\overline{\omega}_{\boldsymbol{\nu}}$ proves to fall into the above indicated interval around the $\boldsymbol{\mu}$-th resonance (we assume the condition $\sigma r_c/R \ll 1$ of weak intramode scattering to be met) its contribution to the shift and the width of the $\boldsymbol{\mu}$-th resonance is estimated as

$$\omega_{\boldsymbol{\mu}} - \omega_{\boldsymbol{\mu}0} = \frac{\sigma^2 \lambda A_{\boldsymbol{\mu}\boldsymbol{\nu}}(\omega_{\boldsymbol{\mu}0} - \overline{\omega}_{\boldsymbol{\nu}})}{(\delta\omega_{\boldsymbol{\mu}0} + \sigma\lambda)^2} , \qquad (31a)$$

$$\delta\omega_{\boldsymbol{\mu}} = \frac{\sigma^2 \lambda A_{\boldsymbol{\mu}\boldsymbol{\nu}}}{\delta\omega_{\boldsymbol{\mu}0} + \sigma\lambda} . \qquad (31b)$$

Here $\omega_{\boldsymbol{\mu}0}$ and $\omega_{\boldsymbol{\mu}}$ are the cyclic spectral frequency of empty resonator and that of the resonator filled with random inhomogeneities, respectively, $\delta\omega_{\boldsymbol{\mu}0}$ and $\delta\omega_{\boldsymbol{\mu}}$ are relative widths of their spectral lines, the parameter $\lambda \sim r_c/R$. If several resonances fall into the indicated vicinity of the $\boldsymbol{\mu}$-th resonance simultaneously, they contribute additively to both the level position and width.

The above presented theory shows that the filling of a quasioptical cavity resonator with randomly distributed inhomogeneities results in both the random shift of its resonance lines and the increase of their linewidths. It should be noted that formulas Eqs.(29) and (30) were obtained with the use of some procedure of averaging over the ensemble of macroscopically identical resonance systems whose microscopic realizations are different. What this formally means is that the statistical theory can give only qualitative predictions for the experiment. The situation seems to be quite similar to that one faces when studying problems of wave scattering by randomly rough surfaces [2]. Yet the problem of oscillations in the quasi-optical cavity resonator filled with bulk random inhomogeneities differs substantially from the problem of wave

scattering by randomly rough surface. The point is that oscillations in cavity resonators become established as a result of multiple transmissions of waves (with multiplicity of the order of Q) through the random infill of the resonator. With this consideration in mind one might expect that under the restriction $r_c \ll R$ the characteristics of resonance lines should be effectively self-averaged [24]. If this is the case, the agreement between theory and experiment may be achieved either in the course of small number of trials or even for one particular realization of the random system. The confirmation or the denial of this assumption may be achieved through the comparison of theoretical predictions and the data of corresponding measurements.

D. Numerical simulation

To compare the theory and experiment we calculate the spectrum of TE modes in the quasi-optical cylinder cavity millimeter wave resonator filled with random dielectric inhomogeneities using our theoretical results. At the resonator spectrum calculation the resonator parameters were taken according to the experiment conditions. The spectrum was calculated by solving the excitation task with pointed external dipole. Fig.2 shows the influence of random bulk inhomogeneities on the spectrum at the different values of the parameter σ calculated using Eq.(30). As an example we selected the spectrum in the frequency interval of 36-37 GHz.

It is shown that the spectrum essentially changed at the presence of dielectric inhomogeneities in the resonator. The magnitude of shifting and broadening of spectral lines are caused by resonator filling by inhomogeneities. The solitary spectral lines even at presence of inhomogeneities keep high quality factor, and their broadening is enough small (for example the spectral lines in the range of 36.2-36.6 GHz). At the same time the shifting and broadening of adjacent lines are significant (for example, the spectral lines near of 36.8 GHz). This results that the number of excited lines with high quality factor are essentially decreased if the number of inhomogeneities are getting bigger. The reduction of the number of high quality lines we can explain as "rarefaction" of the resonator spectrum caused by unequal scattering conditions of different modes on inhomogeneities. Fig.3 shows the amplitude-frequency dependence of the resonator spectrum built as the frequency dependence of Green function module at different magnitude of parameter σ. So, at the increase of the number of inhomogeneities in the resonator the adjacent resonances overlapped and the quality factor of combined resonances is getting smaller (for example, spectral lines near by 36.8 GHz).
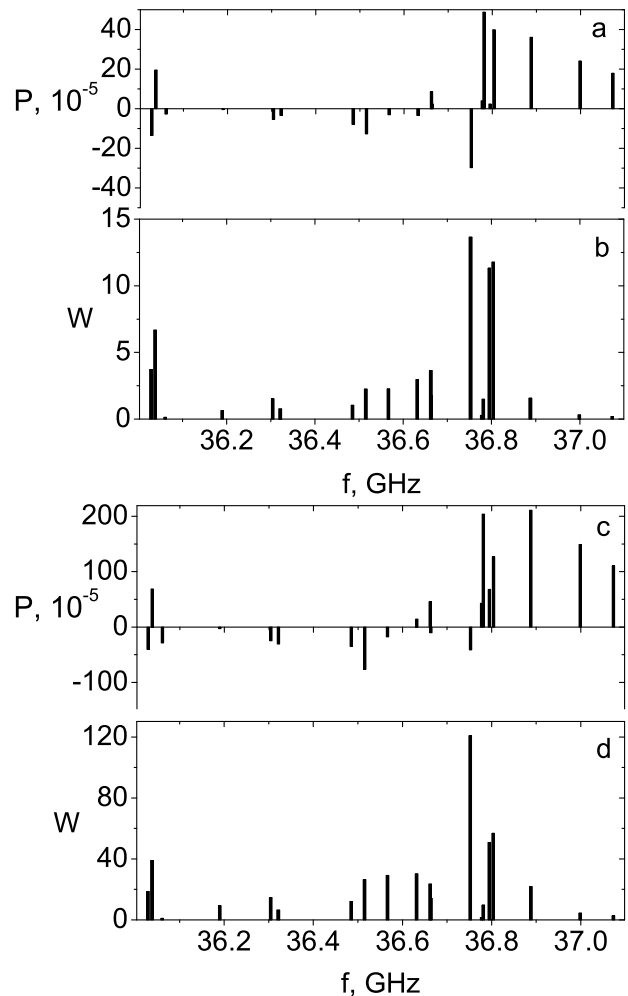

FIG. 2: The dependence of relative shifting $P = \Delta\kappa_\mu^2/\kappa_\mu^2 = \Delta f_\mu^2/f_\mu^2$ (a,c) of spectral lines of the resonator on frequency f caused by bulk inhomogeneities and their relative broadening $W = \tau_d/\tau_\mu^{(ch)} = Q_d/Q_\mu$ (b,d) at the permittivity dispersion values of inhomogeneities: $\sigma = 0.02$ (a,b) and $\sigma = 0.05$ (c,d); $r_c = 0.3$ cm.

III. EXPERIMENT AND DISCUSSION

The main goal of our experiment is the verification of the numerical simulation results of shifting and broadening of spectral lines caused by inhomogeneities and the possibility of resonator spectrum "rarefaction". The another goal is as follows. In the paper [3] we detected strong spectrum stochastization caused by inhomogeneities such as anisotropic sapphire particles with the dimensions of order of operating wavelength placed into the cavity resonator. The spectrum has mixed state due to both regular and chaotic spectrum part exist. In contrast to [3], in the present paper we study the influence on the resonator spectrum relatively small isotropic inhomogeneities. Such inhomogeneities can be made by styrofoam particles with the average value of the permittivity closed to one and with small dielectric loss angle.
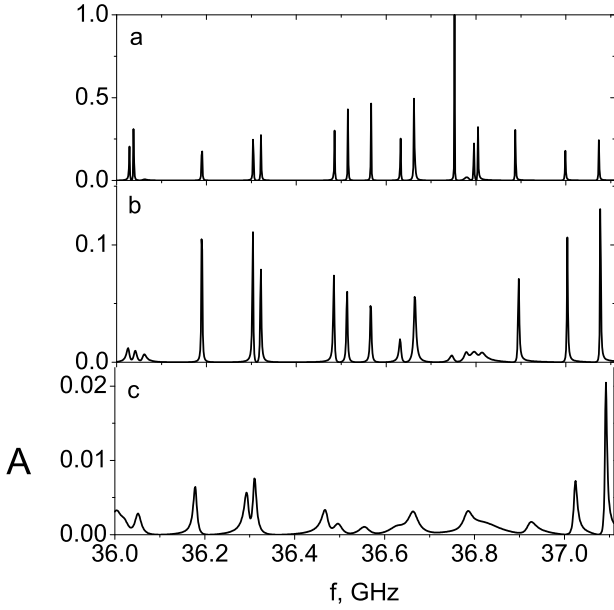
FIG. 3: The amplitude-frequency dependence of the empty resonator spectrum (a) and the spectrum of resonator with inhomogeneities at $\sigma = 0.02$ (b) and $\sigma = 0.05$ (c); $r_c = 0.3$ cm. The amplitude is normalized by maximal amplitude of resonances for the empty resonator in the considered wave range.
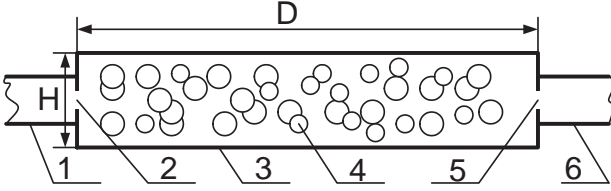

FIG. 4: The quasi-optical cylinder cavity millimeter wave resonator filled with inhomogeneities, 1,6 are input/output waveguides, 2,5 are holes coupling with waveguides, 3 is the resonator body, 4 is styrofoam particles, D is the resonator diameter, H is its hight; $D = 130$ mm, $H = 14$ mm.

Thus, there is a question: is it possible the spectrum chaotization here?

A. Experiment technique

The study of the influence of inhomogeneities on the cavity resonator spectrum we carried out at frequency range of 32-37 GHz. We took a quasi-optical cylinder millimeter wave resonator random filled with styrofoam particles (Fig.4). The styrofoam particles have the real part of permittivity is about one, close to the open-air permittivity and small dielectric loss, $\varepsilon = 1.04 + i10^{-4}$.

To study the influence of small dielectric particles on the resonator spectrum it is necessary to provide high quality factor for the resonator oscillations without inhomogeneities. For that we excited TE mode. The mag-

netic field vector of this mode is directed along the resonator z-axis, and microwave currents do not cross the interface between the flat resonator face and cylinder surface. Owing to that the empty resonator has high quality factor up to 2×10^4 . To excite the selected mode we used a waveguide diffraction antenna. It is the circular hole with the diameter of 2 mm in a thin diaphragm with the thickness of 0.1 mm closing the input waveguide. The diaphragm surface is flush-mounted with the side-cut cylinder surface of the resonator. The same antenna is used to receive the oscillations on the opposite side of the cylinder surface.

The resonator spectrum was detected using "on pass" regime in 32 - 37 GHz wave range by the wide frequency standing wave ratio meter. Measurement process was automated. Signal from the measurement device using analog-digital conversion goes into computer. The further signal processing (the determination of the spectral line intensity, its quality factor, and frequency) was done by special software GUI application. It gives resource in a short space of time to handle measurement data for a huge number of realizations of the random inhomogeneities distribution in the resonator. Owing to that the frequency and quality factor measurement accuracy for spectral lines do not exceed 0.1% and 1%, respectively. The styrofoam particles that are used as inhomogeneities have the size about of 2-3 mm. The space distribution of them was arbitrary for each realization. The spectral characteristics were measured depending on the number of these inhomogeneities.

B. Shifting and broadening of spectral lines. Effect of spectrum "rarefaction"

The spectrum of the empty resonator is dense enough and consists of 84 narrow spectral lines in the range of 32-37 GHz (Fig.5). Each line was identified according to mode indexes of corresponding to its eigen resonator oscillation. We found out the oscillations with high quality factor (Q) of order of 10^4 and higher have small azimuth indexes $(n \sim 1)$ and high radial indexes $(l >> 1)$. For the base oscillation mode $(n >> 1, l = 1)$ that has the field distribution concentrated inside of the resonator side-cut (whispering-gallery oscillations [5]), Q is about $2 \cdot 10^3$. The spectrum of the resonator with inhomogeneities is essentially different from the spectrum of the empty one. By the increase of the number of inhomogeneities spectral lines are getting wider, and Q is getting smaller. The intensity of broadening lines is decreased as well. The spectrum keeps only few lines with enough high intercity and Q value. For such lines the Q value is close to the Q-factor of the spectral lines in the empty resonator. So, the number of spectral lines with enough high intensity and with $Q > 10^4$ is 52 in the empty resonator (Fig.5a), at filling the resonator by styrofoam particles the number of such lines is 10 (Fig.5b), and at filling the resonator by pressed styrofoam particles
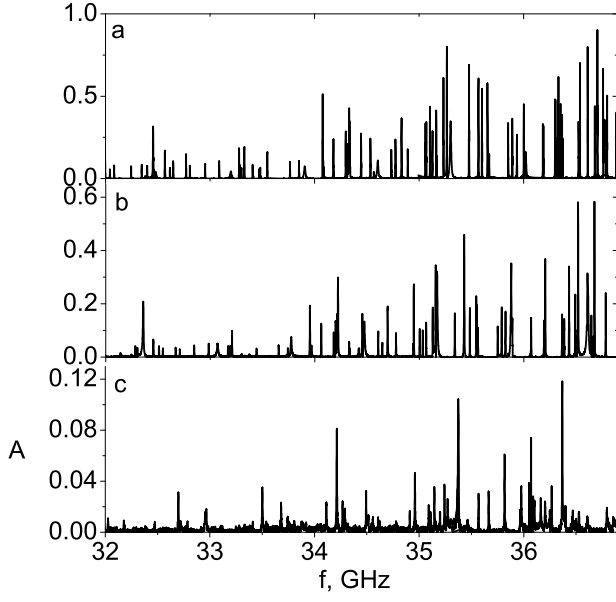
FIG. 5: The spectrum of the empty resonator (84 spectral lines) (a); the resonator filled with styrofoam particles (77 spectral lines) (b); the resonator filled with pressed styrofoam particles (57 spectral lines) (c); The normalization of amplitude was made on the maximal amplitude value for the empty resonator.

the number of lines is 3 (Fig.5c).

The presence in the resonator with inhomogeneities together with broadening lines few high Q spectral lines is equivalent to the spectrum "rarefaction" (Fig.5). It is necessary to note that the presence of lines with high Q indicates that the broadening is specified, mainly, by the inhomogeneous of the resonator filling permittivity and is not specified by additional dissipation loss caused by small dielectric loss in the styrofoam. Except broadening of spectral lines they have frequency shifting. This frequency shifting has both regular component and random one as well (Fig.6).

The regular shifting happens towards the low frequency direction because of the increase of average dielectric permittivity of inhomogeneous medium in the resonator. For example, it is 150 MGz for the resonator fully filled with styrofoam particles.

It is significant that if the distance between resonances is enough small they can both come close and depart from each other because of influence of inhomogeneities. Their Q is essentially changed (Fig.7,8): it can increase for one of them and decrease for another.

Both the spectral lines broadening and their shifting have quality agreement with described above theory and can be interpreted in the terms of intermode scattering. The maximal broadening is for the adjacent lines that is agreed with theory.

We can give the following physical explanation of spectral lines broadening caused by random inhomogeneities based on quantum mechanics interpretation of resonator
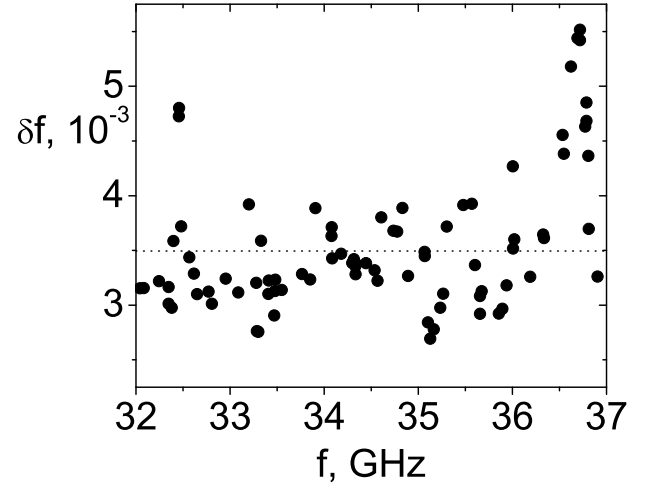

FIG. 6: The dependence of the frequency shifting $\delta f = (f_{empty} - f_{in\,hom})/f_{empty}$ on frequency f for the resonator filled by inhomogeneities. The dotted line is the value of shifting regular component.
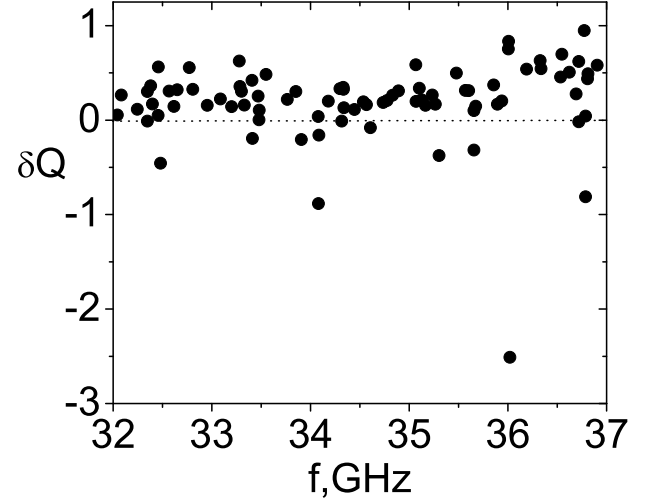

FIG. 7: The relative Q-factor deviation $\delta Q = (Q_{empty} - Q_{in\,hom})/Q_{empty}$ depends upon frequency f for the resonator filled by inhomogeneities. For major part of resonances the relative Q-factor deviation is more than zero, i.e., their Q is less than for the empty resonator. The relative Q-factor deviation is less than zero for several nearest adjacent resonances that have overlapped spectral lines (in the case of the empty resonator) and split into separate resonances at the presence of inhomogeneities, i.e., the spectral line repulsion takes place here (see Fig.8).

oscillations. In empty resonator the lifetime of a resonant mode in a given state (describing by an eigen resonant frequency) and its spectral line width define by dissipation loss in the resonator body. The uncertainty of mode energy within the spectral line width is caused by this loss. The intermode interaction appears at inserting inhomogeneities into the resonator. Owing to that the uncertainty of mode energy and, correspondingly, the
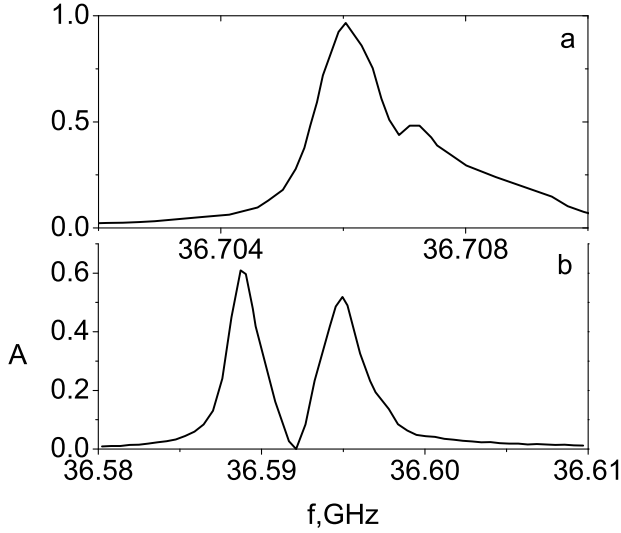
FIG. 8: The adjacent resonances with indexes n = 34, l = 3; n = 4, l = 15; in the empty resonator (a) and in the resonator filled by inhomogeneities (b). The normalization of the spectral lines amplitude was made on the maximal spectral line amplitude for the empty resonator.

spectral line width increase due to the mode transition between nearest states. The mode energy loss due to dissipation loss in the resonator body and transition to the neighboring state are additive to the spectral line width.

C. Statistical analysis of IF intervals

If the number of inhomogeneities increases the spectrum of the quasi-optical resonator possesses stochastic character that is visualized in distribution of IF intervals. In order to define the relation between regular and random spectral components the comparison of IF intervals distribution obtained experimentally with different theoretical distributions based on a priori data about statistical process is usually used. In particular, we use the Brody function determined the distribution of IF intervals probability $P_B(s)$ that is given by

$$P_B(s) = As^{\beta} \exp(-Bs^{1+\beta}) \qquad (32)$$

where $s = (\omega_n - \omega_{n-1})\rho(\omega_n)$, ω_n is the spectral line frequency, $\rho(\omega_n)$ is the spectral density - the superposition of regular and random motion density, β is the measure of stochastic motion, constants A and B are defined from the condition of standardization: $A = (1+\beta)B$, , $B = \Gamma^{1+\beta}(2+\beta)(1+\beta)^{-1}$, $\Gamma(z)$ is the Gamma function. At $\beta \to 0$ IF intervals in the spectrum are not correlated and can be described by the Poisson distribution, and at $\beta \to 1$ we have Wigner distribution, when the repulsion effect of spectral lines exists, that is the probability of closest to zero inter-frequency interval is equal zero as well.

If stochastization measure in the spectrum is relatively small and the distribution of IF intervals is close to the Poisson one we can use Berry-Robnik distribution $P_{BR}(s)$ [26] that is given by

$$P_{BR}(s) = \rho^2 e^{-\rho s} erfc\left(\frac{\sqrt{\pi}}{2}(1-\rho)s\right) + \left[\frac{\pi}{2}(1-\rho)^2 s + 2\rho\right](1-\rho)e^{-\rho s - \frac{\pi}{4}(1-\rho)^2 s^2} \qquad (33)$$

where $erfc(x) = \frac{2}{\sqrt{\pi}}\int\limits_x^{\infty} \exp(-t^2)dt$.

$\rho = 1$ is relative phase volume occupied with regular trajectory in mixed systems. The limit $\rho \to 1$ corresponds to regular system; $\rho \to 0$ is completely chaotic system. $1 - \rho$ is relative phase volume occupied with chaotic motion.

The experimental data show that in the empty resonator the IF intervals distribution is close enough to the Berry-Robnik distribution with $\rho = 1$ (Poisson distribution, $P(s) \sim \exp(-s)$)(Fig.9a). By increase of the number of inhomogeneities (styrofoam particles) the function $P(s)$ is reduced and it has maximum at small s. The presence of the maximum $P(s)$ indicates the appearance of the repulsion effect of spectral lines, and the random component is increased. We found out that at full filling the resonator by styrofoam particles $\rho = 0.4$ (Fig.9b).

As we can see in Fig.9 the function $P(s)$ has maximum at small s that is not described by the Berry-Robnik dependence [26]. The presence of such a maximum can be explained by Chaos-Assisted Tunneling (CAT) [27] in the resonator with random inhomogeneities. The proposed in [27] IF distribution function gives the possibility to describe the observed distribution at the parameter $\rho = 0.4$ corresponding to the classical dynamics in the system and the parameter $\nu = 0.1$ describing the tunneling between different states of modes.

We calculated the spectral rigidity for the resonator with inhomogeneities. The spectral rigidity $\Delta_3(L)$ is an integral characteristic of degree of spectral lines ordering for frequency distances that is much more than interfrequency interval. It is given by [15]

$$\Delta_3(x, L) = \frac{1}{L} \min_{A,B} \int\limits_x^{x+L} [n(\varepsilon) - A\varepsilon - B]^2 d\varepsilon \qquad (34)$$

where L is the interval length on which the function $\Delta_3(x, L)$ is determined. The function $n(\varepsilon)$ is built as follows [15]. For the sequence of the frequencies ω_n normalized on unit density ($\omega_n = \omega_{n-1} + S_n$), we introduce a staircase function $n(\varepsilon)$ equals the number of frequencies with $\omega_n < \varepsilon$.

The function $n(\varepsilon)$ has a staircase view with average unit tilt. The function $\Delta_3(x, L)$ is determined as the minimum of quadratic deviation $n(\varepsilon)$ in the interval $(x, \ x + L)$ from the straight line. The meaning of the averaged in x spectral rigidity (Eq.(34)) $\langle\Delta_3(x, L)\rangle_x$ depends only on L and is denoted as $\Delta_3(L)$.
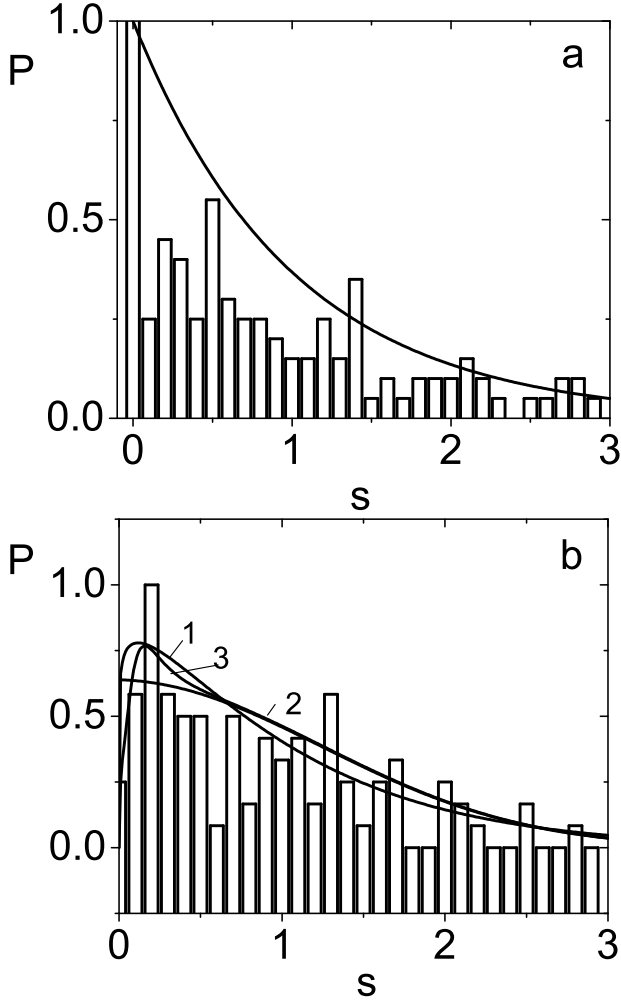
FIG. 9: The distribution of IF intervals, $P(s)$. Figure 9a is for the empty resonator spectrum. Solid line is for the Berry-Robnik distribution at $\rho \rightarrow 1$ or the Brody distribution at $\beta \rightarrow 0$. Figure 9b is for the spectrum of the resonator fully filled with styrofoam particles. The curve 1 is for Brody distribution at $\beta = 0.1$; the curve 2 is for Berry-Robnik distribution at $\rho = 0.4$; the curve 3 is for Podolskiy-Narimanov distribution at $\rho = 0.4$ and $\nu = 0.1$ [27].
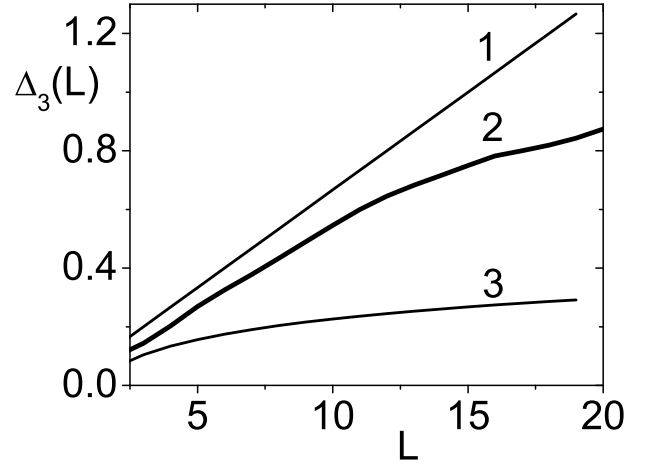

FIG. 10: The spectral rigidity for the resonator filled with styrofoam particles (curve 2). The straight line is for the spectral rigidity of Poisson distribution (curve 1), curve 3 is for GOA.

IV. ACTIVE RESONATOR WITH RANDOM INHOMOGENEITIES

We studied also an active quasi-optical resonator with random inhomogeneities. In comparison with the considered above passive resonator the active one is a self-sustained oscillation system where the presence of random inhomogeneities affects on the excitation of resonator oscillations. For that we used the same resonator as mentioned above (Fig. 4) with pointed Gunn diode inside. The microwave electrical field for TE mode was directed along diode's axis. The diode DC power supply was implemented through a filter as a quarter-wave microwave isolator; owing to spurious microwave radiation was prevented.

Quasi-optical resonator with Gunn diode is an active oscillator with distributed parameters. Near threshold of excitation in such an oscillator with the empty resonator was detected unstable multi-frequency generation. Such kind of generation we can explain by frequency jumps between adjacent spectral lines with high quality factor. If excitation threshold was highly exceeded, as a result of frequency competition, corresponding to these spectral lines, mono-frequency generation occurs (Fig.11b). The active oscillator selects "itself" the only frequency to provide maximal regeneration factor [28].

The random inhomogeneities lead to effective rarefaction of the resonator spectrum. The number of adjacent spectral lines with high quality factor is reduced, and, as a result, multi-frequency generation disappears. At small exceeding of the generation threshold the noise generation appears (Fig.11c) in the resonator with inhomogeneities. At big exceeding of the generation threshold the stable mono-frequency is observed (Fig.11d). Owing to inhomogeneities mono-frequency generation possesses greater frequency stability, and the number of generat-

The obtained curve $\Delta_3(L$ for the resonator with random inhomogeneities is shown in Fig.10. This curve (2) is placed between the spectral rigidity for the Poisson distribution (curve 1, $(\Delta_3(L) = L/15)$)) and curve 3 that is corresponded to the spectral rigidity for Gauss orthogonal ensemble (GOA) $\Delta_3(L) = 1/\pi^2 \ln L - 0.00687$ [15]. The latter is observed at the modeling of quantum chaos in microwave cavity unstable resonators similar to Sinai and Bunimovich billiards [14].
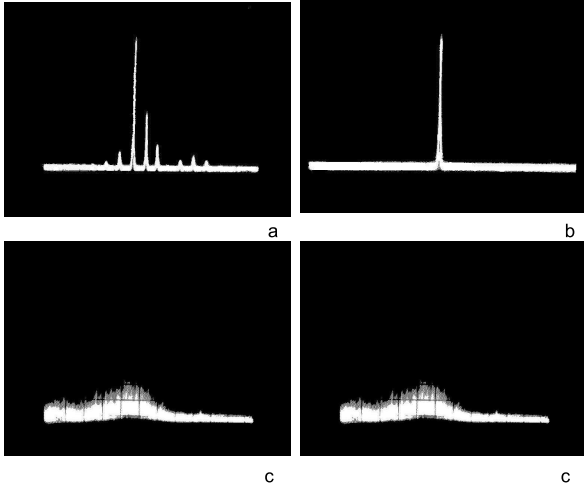
FIG. 11: Oscillogram of the 36 GHz generation obtained by a millimeter wave spectrum analyzer. (a) and (b) are for the empty resonator; multi-frequency generation near the generation threshold (a) and mono-frequency generation much far from the generation threshold (b). (c) and (d) are for the resonator filled with inhomogeneities; chaotic generation near the generation threshold (c) and mono-frequency stable generation much far from the generation threshold (d).
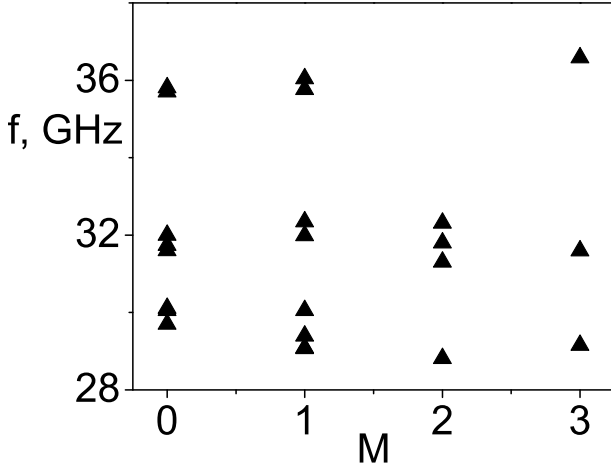

FIG. 12: The generating frequencies f depending on the number of inhomogeneities M. The empty resonator ($M = 0$), the resonator quarter-filled with inhomogeneities ($M = 1$), the resonator half-filled with inhomogeneities ($M = 2$), the resonator fully filled with inhomogeneities ($M = 3$).

ing frequencies is reduced in the range of diode negative resistance. Fig.12 shows generating frequencies for the empty resonator and with random bulk inhomogeneities one.

V. CONCLUSION

The statistical spectral theory of quasi-optical cavity resonator filled with random dielectric inhomogeneities was developed in the present paper. We showed that the presence of inhomogeneities leads to the broadening and shifting of spectral lines. It is found out that the nature of broadening and shifting of spectral lines is relevant to the intermode scattering. The scattering effect for the given spectral line essentially depends on frequency distance between of it and adjacent ones and is sharply decreased for bigger distances. Under the influence of ransom inhomogeneities original spectrum modification occurs that can be interpreted as spectrum rarefaction. The spectrum is rarefied because of solitary spectral lines are not practically subjected to the influence of inhomogeneities. The quality factor of such lines and, correspondingly, their intensity stay high at the resonator excitation. The intensity of adjacent lines broadened under the influence of inhomogeneities is essentially reduced. Owing to that at the great number of inhomogeneities the resonator spectrum is "rarefied" - few solitary high quality factor spectral lines prevail in the spectrum. Theoretical prediction of broadening and shifting of resonator spectral lines and spectrum "rarefaction" are subject to experimental check.

For that purpose we studied experimentally in 8-millimeter wave range the spectrum of the quasi-optical cavity resonator filled with random small-scattered bulk inhomogeneities. These inhomogeneities were styrofoam particles with smaller size than operating wavelength. It is found out that such inhomogeneities lead to broadening and shifting the spectral lines. As experiment showed the maximum of their influence was on frequency adjacent spectral lines. The solitary lines, according to our theory, were subjected to this influence in much smaller degree. We detected the effect of stochastic spectrum "rarefaction". They prove that the main mechanism of broadening and shifting of spectral lines is relevant to inhomogeneities intermode scattering. We studied also chaotic properties of oscillations in our resonator. It is found out that the empty resonator has IF intervals distribution similar to Poisson distribution that is typical to the spectrum with non-correlated IF intervals. Even at small number of inhomogeneities the resonator spectrum has random part that increases in proportion to the number of inhomogeneities in the resonator. At that IF intervals distribution in random inhomogeneous resonator is described by Brody and Berry-Robnik distributions of IF intervals.

We obtained the results concerning the influence of bulk inhomogeneities on the process of generation in a self-oscillatory system. The self-oscillatory system was a quasi-optical millimeter wave cavity resonator containing inhomogeneities with an active element as Gunn diode. We detected that inhomogeneities leads to the essential "rarefaction" of the spectrum and creates conditions for monochromatic stable generation in self-oscillatory system. The inhomogeneous quasi-optical cavity millimeter wave resonator (passive and active) can serve as a model of semiconductor quantum billiard. Based on our results we suggest using such billiards with spectrum rarefied by

random inhomogeneities as an active system of semiconductor laser.

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